

Name \_\_\_\_\_

Q1 Test 3 Review Sheet

- Write the equations of the tangent line to the curve  $f(x) = -\sin(x)$  when  $x = \pi$ .
- Given  $h(x) = (3x^3 - x^2 + 10x + 2)\cos(x)$ , find  $h'(x)$ .
- Find the derivative of the following function in simplest form:  $y = \frac{3x^2 - 2}{2x - 3}$

For questions 4 – 7 use the following table to find  $y'$  at  $x = 1$ , if:

$f(1)$	$f'(1)$	$g(1)$	$g'(1)$
3	4	1	-2

4.  $y = f(x)g(x)$   $y' = f(x) \cdot g'(x) + f'(x) \cdot g(x)$

$$= (3)(-2) + (4)(1) = \boxed{-2}$$

5.  $y = \frac{f(x)}{g(x)}$

$$y' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2} = \frac{(1)(4) - (3)(-2)}{1^2} = \boxed{10}$$

6.  $y = x^4 g(x)$

7.  $y = \frac{x^3 - 2x}{g(x)}$

8. Find the coordinates of the point(s) where  $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$  has horizontal tangents.

9. Find the equation of the tangent line to the curve,  $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$  when  $x = -2$

10. Find  $f'(x)$  if  $f(x) = \frac{3x^2}{x-1}$

11. Find  $f'(x)$  if  $f(x) = 3x^2 \sin x$

Find the derivative of each of the following:

12.  $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

13.  $f(x) = \sin(3x+1)$

14.  $f(x) = \sqrt[4]{(x^2+5x)^3}$

15.  $y = \ln x^5$

16.  $y = e^{4x^3+2}$

17.  $y = x^5 - \ln(x) + 5e^2$

$$15) y = \ln(x^5)$$

$$\frac{dy}{dx} = \frac{1}{x^5} \cdot 5x^4 = \frac{5x^4}{x^5} = \boxed{\frac{5}{x}}$$

$$16) y = e^{4x^3+2}$$

$$\frac{dy}{dx} = e^{4x^3+2} \cdot 12x^2 = \boxed{12x^2 e^{4x^3+2}}$$

$$17) y = x^5 - \ln(x) + \underbrace{5e^2}_{\text{constant}}$$

$$\frac{dy}{dx} = 5x^4 - \frac{1}{x} + \cancel{5e^2 \cdot (0)}$$

$$\frac{dy}{dx} = \boxed{5x^4 - \frac{1}{x}}$$

$$6) y = x^4 \cdot g(x)$$

$$\frac{dy}{dx} = x^4 \cdot g'(x) + 4x^3 \cdot g(x)$$

$$= x^4(-2) + 4x^3(1)$$

$$\frac{dy}{dx} = -2x^4 + 4x^3$$

$$\begin{aligned} \textcircled{*} \quad &= -2(1)^4 + 4(1)^3 \\ X=1 \quad &= -2 + 4 \\ &= \boxed{2} \end{aligned}$$

$$7) y = \frac{x^3 - 2x}{g(x)}$$

$$\frac{dy}{dx} = \frac{g(x) \cdot (3x^2 - 2) - (x^3 - 2x) \cdot g'(x)}{(g(x))^2}$$

$$= \frac{(1)(3(1)^2 - 2) - (1^3 - 2(1)) \cdot (-2)}{1^2} = \frac{1 - 2}{1} = \boxed{-1}$$

1)  $f(x) = -\sin(x)$    $x = \pi$

Point: use the function

$$f(\pi) = -\sin(\pi)$$

$$f(\pi) = 0$$

$$(\pi, 0)$$

Slope: use the Derivative

$$f'(x) = -\cos(x)$$

$$f'(\pi) = -\cos(\pi)$$

$$f'(\pi) = -(-1)$$

$$f'(\pi) = 1$$

$$\text{Slope} = 1$$

$$y - 0 = 1(x - \pi)$$

$$\boxed{y = x - \pi}$$

8)  $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

$$f'(x) = x^3 - x^2 - 2x$$

$$0 = x^3 - x^2 - 2x$$

$$= x(x^2 - x - 2)$$

$$= x(x-2)(x+1)$$

$$x=0 \quad x=2 \quad x=-1$$

$$f(0) = \frac{1}{4}(0)^4 - \frac{1}{3}(0)^3 - (0)^2$$

$$= 0$$

$$(0, 0)$$

$$f(2) = \frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - 2^2$$

$$= 4 - \frac{8}{3} - 4$$

$$= -\frac{8}{3}$$

$$(2, -\frac{8}{3})$$

$$f(-1) = \frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - (-1)^2$$

$$= \frac{1}{4} + \frac{1}{3} - 1$$

$$= -\frac{5}{12}$$

$$(-1, -\frac{5}{12})$$

Product rule!

$$2) \quad h(x) = (3x^3 - x^2 + 10x + 2)(\cos(x))$$

$$h'(x) = (3x^3 - x^2 + 10x + 2)(-\sin(x)) + (9x^2 - 2x + 10)(\cos(x))$$

$$3) \quad y = \frac{3x^2 - 2}{2x - 3} \quad \leftarrow \text{Quotient}$$

$$\frac{dy}{dx} = \frac{(2x-3)(6x) - (3x^2-2)(2)}{(2x-3)^2} =$$

$$= \frac{12x^2 - 18x - 6x^2 + 4}{4x^2 - 12x + 9} = \boxed{\frac{6x^2 - 18x + 4}{4x^2 - 12x + 9}}$$

8) Horizontal tangents when derivative = 0

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$$

$$f'(x) = x^3 - x^2 - 2x$$

$$0 = x^3 - x^2 - 2x$$

$$= x(x^2 - x - 2)$$

$$= x(x-2)(x+1)$$

$$x=0 \quad x=2 \quad x=-1$$

$$f(0) = 0$$

$$f(2) = -\frac{8}{3}$$

$$f(-1) = -\frac{5}{12}$$

$$(0, 0)$$

$$(2, -\frac{8}{3})$$

$$(-1, -\frac{5}{12})$$

9)  $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$

Point:

$$f(-2) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$$

$$f(-2) = \frac{8}{3}$$

$$(-2, \frac{8}{3})$$

Slope:

$$f'(x) = x^3 - x^2 - 2x$$

$$f'(-2) = -8 - 4 + 4$$

$$f'(-2) = -8$$

$$y - \frac{8}{3} = -8(x + 2)$$

10)  $f(x) = \frac{3x^2}{x-1}$

$$f'(x) = \frac{(x-1)(6x) - (3x^2)(1)}{(x-1)^2}$$

$$= \frac{6x^2 - 6x - 3x^2}{x^2 - 2x + 1} = \frac{3x^2 - 6x}{x^2 - 2x + 1}$$

$$11) f(x) = 3x^2 \cdot \sin(x)$$

$$f'(x) = (3x^2)(\cos(x)) + (\sin(x))(6x)$$

$$12) f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$$

$$f(x) = 5x + 2x^{1/3} - 3x^{-2}$$

$$f'(x) = 5 + \frac{2}{3}x^{-2/3} + 6x^{-3}$$

$$= 5 + \frac{2}{3\sqrt[3]{x^2}} + \frac{6}{x^3}$$

$$13) f(x) = \sin(3x+1) \quad \swarrow \text{chain Rule}$$

$$f'(x) = \cos(3x+1) \cdot (3)$$

$$= 3 \cos(3x+1)$$

$$14) f(x) = \sqrt[4]{(x^2+5x)^3} =$$

$$f(x) = (x^2+5x)^{3/4}$$

$$f'(x) = \frac{3}{4}(x^2+5x)^{-1/4} \cdot (2x+5)$$

$$f'(x) = \frac{3(2x+5)}{4(x^2+5x)^{1/4}} = \frac{6x+15}{4\sqrt[4]{x^2+5x}}$$

Extra:23) Given  $f(x) = \sqrt[7]{x^5 + 3x^4}$  find  $f'(2)$ 

$$f(x) = (x^5 + 3x^4)^{1/7}$$

$$f'(x) = \frac{1}{7} (x^5 + 3x^4)^{-6/7} \cdot (5x^4 + 12x^3)$$

$$f'(x) = \frac{1(5x^4 + 12x^3)}{7(x^5 + 3x^4)^{6/7}} = \frac{(5x^4 + 12x^3)}{7\sqrt[7]{(x^5 + 3x^4)^6}}$$

$$f'(2) = \frac{1}{7} (2^5 + 3(2)^4)^{-6/7} (5(2)^4 + 12(2)^3)$$

$$f'(2) = .58775$$